Conductance of the capacitively coupled single-electron transistor with a Tomonaga-Luttinger liquid island in the Coulomb blockade regime

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We consider the electron transport in the capacitively coupled single-electron transistor with an ultrasmall Tomonaga-Luttinger liquid island. The charging effects, as well as the Tomonaga-Luttinger liquid nature, are treated by a self-consistent theory of the Coulomb blockade using the open boundary bosonization technique. Analytical expressions for conductance are derived in the limits of low and high voltages and temperatures, for bulk and edge island contact geometries, and for arbitrary environmental impedance. For an infinite system, we obtain the power law of the conductance with the exponent changed from the usual Tomonaga-Luttinger exponent due to the effects of the electromagnetic environment. For a finite system, we obtain expressions for the conductance as a function of voltage near the Coulomb blockade boundary and as a function of temperature for low temperatures; these expressions differ from the usual power-law behavior. The results show the potential for improving the accuracy of single-electron devices such as those used in electrical metrology.

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I. INTRODUCTION

Developments in fundamental metrology over recent years have made it feasible to consider introducing a new International System (SI) of Units based on a set of exactly defined values of fundamental constants.[1] One requirement is the ability to realize the base units in a straightforward fashion. For example, by fixing the value of the elementary charge e, the ampere could be realized from the simple relation $I=ef$ by using a single-electron pump that transfers individual electrons at a driving frequency f traceable to the SI second via an atomic clock. In order to be suitable for practical applications, such a pump should be able to generate currents of the order of 1 nA with an accuracy of the order of 10^{-8}. A variety of devices and materials have been investigated to satisfy these requirements. These include semiconducting,[2] normal-metal,[3] and superconducting[4] devices, as well as hybrid devices based on normal-metal-superconductor junctions[5] or surface acoustic wave-induced pumping through a carbon nanotube.[6]

Another important application of single-electron devices in regard to the new SI is the test based on quantum metrological triangle[7] of the assumed exactness of the expressions for the Josephson constant $K_J=h/2e$ and the von Klitzing constant $R_K=h/e^2$. At present, the device used for this purpose[8] is the so-called R-pump.[9] This device, as proposed in Ref. 10, is based on the effect of the circuit-impedance-induced power-law suppression of the cotunneling processes that limit the accuracy of the single-electron-tunneling devices.[11] The conductance of a capacitively coupled single-electron-tunneling transistor (C-SET) in the Coulomb blockade (CB) regime behaves as $G\sim V^{2+2\zeta}$, where $\zeta=R/R_K$, and R is the zero-frequency impedance of the environment. In this case, the width of the electrodes and the island are much larger than the Fermi wavelength, making a large number of transverse channels available for the tunneling electron. Since the relevant energies are close to the Fermi level, Landau's Fermi-liquid (FL) theory is applicable and the only trace of electron-electron interaction is described by the charging energy of the island. The above power law is obtained from the tunnel Hamiltonian of the form $H_T=\Sigma_{k,l}T_{kl}c_k^{\dagger}c_l e^{i\phi}$, where the phase ϕ is a linear combination of Bose operators corresponding to the electromagnetic environment modes. Using the fluctuation-dissipation theorem, the phase-phase correlation function can be expressed in terms of the environmental impedance from whence the above power law of conductance is derived.

On the other hand, when the number of transverse channels in the island is reduced to one, the Fermi-liquid description of nearly free quasiparticles is no longer applicable (for a review, see, for example, Ref. 12). The electron-electron interaction now has a drastic effect, resulting in a charge-spin separation. However, the fermion field Ψ can be represented in terms of collective charge and spin bosonic fields Φ as $\Psi\sim e^{i\Phi}$. By applying the Baker-Hausdorff formula and the cumulant expansion for bosonic modes, the Green's function of the electron can be expressed as a function of the $\Phi-\Phi$ correlators. Similarly to the environmental effect outlined above, this also leads to a power-law dependence of conductance versus voltage $G\sim V^{\alpha_c}$ at large biases ($eV\gg k_BT$). In this case the exponent α_c depends on the interaction parameters of electrons in a one-dimensional (1D) channel. This result applies for an infinite length of the channel and has been observed in carbon nanotube measurements.[13] The Green's function in the case of a finite system of length L_d can be obtained by conformal mapping of the complex plane onto a cylinder of radius L_d/π, which results in a scale-dependent exponent.[14] This has also been studied numerically in Ref. 15. In the opposite limit of low voltages ($eV\ll k_BT$), the conductance of the infinite system also shows power-law behavior $G\sim T^{\alpha_c}$. This has been confirmed experimentally[13] but, as the temperature was lowered below

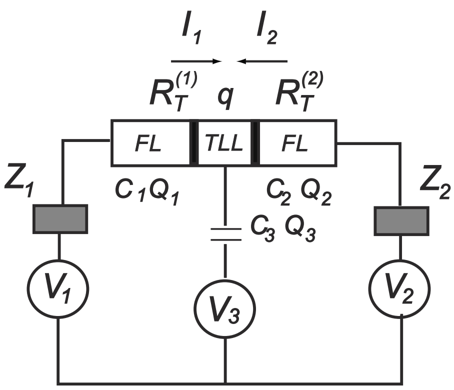

FIG. 1. Model of the system: equivalent electrical circuit for TLL C-SET with environmental impedances $Z_i(\omega)$. TLL island has discrete energy levels with energy spacing ϵ_ν $(\nu=\rho,\sigma)$. ρ and σ denote bosonic excitations of charge and spin degrees of freedom, respectively. There are two kinds of possible contacts between TLL and external electrodes depending on the fabrication process: bulk contact (nanotubes deposited over predefined electrodes) and edge contact (evaporating the electrodes over the nanotubes).

the charging energy of the island, a deviation from the power law was observed. This paper considers conductance in this temperature region, including the environmental effects that are important in metrology as well as in the physics of the Coulomb blockade.

The paper is organized as follows. Section II describes the model and presents the Hamiltonian of the system. Section III derives the expression for the tunneling current. For an infinite system this gives the power law for the conductance as a function of voltage and temperature, with the normal exponent modified by the effect of the electromagnetic environment. For the finite system, we obtain the analytic results for low voltages and temperatures. Section IV summarizes the results and concludes with a proposal for possible applications.

II. MODEL AND HAMILTONIAN OF THE SYSTEM

We consider a voltage-biased Tomonaga-Luttinger liquid (TLL) C-SET connected to an external environment of impedance $Z_i(\omega)$ (Fig. 1). The theoretical framework is similar to a C-SET with FL electrodes as reported in Refs. 16 and 17.The Hamiltonian of the system is given by $\mathcal{H}=\mathcal{H}_0+\mathcal{H}_T$, where $\mathcal{H}_0=\mathcal{H}_{\rm FL}+\mathcal{H}_{\rm TLL}+\mathcal{H}_{\rm em}$, and $\mathcal{H}_T$ is the tunneling Hamiltonian. The terms in $\mathcal{H}_0$ describe the Fermi-liquid electrodes ($\mathcal{H}_{\rm FL}$), the Tomonaga-Luttinger liquid island ($\mathcal{H}_{\rm TLL}$), and the electromagnetic environment ($\mathcal{H}_{\rm em}$).

A. Fermi-liquid electrodes

A description of electrons in all three electrodes can be expressed using the FL model,

$$\mathcal{H}_{\rm FL}=\sum_{i=1}^{3}\sum_{\boldsymbol{k}s}\epsilon^{(i)}(\boldsymbol{k})a_{\boldsymbol{k}s}^{(i)\dagger}a_{\boldsymbol{k}s}^{(i)}, \tag{1}$$

where $\epsilon^{(i)}(\boldsymbol{k})=(\hbar\boldsymbol{k})^2/(2m)$, and $a_{\boldsymbol{k}s}^{(i)\dagger}(a_{\boldsymbol{k}s}^{(i)})$ are the creation (annihilation) operators of the electron with wave vector $\boldsymbol{k}$ and spin s. Indexes $i=1,2,3$ refer to the left, right, and gate electrodes, respectively.

B. Tomonaga-Luttinger liquid island

The TLL island[12,18–20] can be described using the Tomonaga model,[18] which is expressed by the g-ology Hamiltonian[21] in which only the forward-scattering terms g_2 and g_4 are included. Using the open boundary bosonization technique,[22] we start with the fermion field operator,

$$\Psi(x)=\sum_s\Psi_s(x)=\sum_{s,r=\pm}\Psi_{rs}(x)=\sum_{s,r=\pm}e^{irk_Fx}\psi_{rs}(x), \tag{2}$$

and impose the boundary condition for the island of length L_d, $\Psi_s(0)=\Psi_s(L_d)=0$, that is,

$$\psi_{-s}(0)=-\psi_{+s}(0), \tag{3}$$

$$\psi_{-s}(L_d)=-e^{2ik_FL_d}\psi_{+s}(L_d). \tag{4}$$

We introduce the chiral boson phases as

$$\psi_{rs}(x)=\frac{\eta_{rs}}{\sqrt{2\pi\alpha}}e^{i\Phi_{rs}(x)}, \tag{5}$$

$$\Phi_{rs}(x)=\theta_{rs}+\phi_{rs}(x)=\frac{r}{\sqrt{2}}\sum_{\nu=\rho,\sigma}s^{\delta_{\nu\sigma}}\Phi_\nu(x)+\frac{1}{\sqrt{2}}\sum_{\nu=\rho,\sigma}s^{\delta_{\nu\sigma}}\Theta_\nu(x). \tag{6}$$

Here α is a cut-off parameter, η_{rs} are the Majorana fermion operators satisfying $\{\eta_{rs},\eta_{r's'}\}=2\delta_{rr'}\delta_{ss'}$, and ν stands for charge (ρ) and spin (σ) degrees of freedom. Taking into account the open boundary condition, the boson phases are expressed as

$$\Phi_\nu(x)=Q_\nu+\frac{\pi}{L_d}N_\nu x+i\sqrt{K_\nu}\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\sin\frac{n\pi x}{L_d}[\alpha_n^{(\nu)\dagger}-\alpha_n^{(\nu)}], \tag{7}$$

$$\Theta_\nu(x)=-\tilde{Q}_\nu-\frac{1}{\sqrt{K_\nu}}\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\cos\frac{n\pi x}{L_d}[\alpha_n^{(\nu)\dagger}+\alpha_n^{(\nu)}]. \tag{8}$$

The Luttinger parameter K_ν and other quantities appearing in Eqs. (7) and (8) are defined as

$$K_\nu=\sqrt{\frac{\pi v_F+[g_{4\nu}-g_{2\nu}]/\hbar}{\pi v_F+[g_{4\nu}+g_{2\nu}]/\hbar}}, \tag{9}$$

$$g_{i\nu}=\frac{1}{2}[g_{i\parallel}+(-1)^{\delta_{\nu,\sigma}}g_{i\perp}], \tag{10}$$

$$N_\nu=\frac{1}{\sqrt{2}}\sum_{rs}s^{\delta_{\nu\sigma}}N_{rs}=\frac{1}{\sqrt{2}}\sum_{\boldsymbol{k}rs}s^{\delta_{\nu\sigma}}:a_{r,\boldsymbol{k},s}^{\dagger}a_{r,\boldsymbol{k},s}:, \tag{11}$$

$$Q_\nu=\frac{1}{2\sqrt{2}}\sum_{rs}rs^{\delta_{\nu\sigma}}\theta_{rs}, \tag{12}$$

$$\tilde{Q}_\nu = -\frac{1}{2\sqrt{2}}\sum_{rs} s^{\delta_{\nu\sigma}}\theta_{rs}, \quad (13)$$

$a^\dagger_{r,\boldsymbol{k},s}(a_{r,\boldsymbol{k},s})$ being creation (annihilation) operators of electrons in the TLL and $:\cdots:$ denotes the normal product. Noticing that the following commutation relations hold between the operators specifying the bosonic excitations and the zero mode $[\alpha_n^{(\nu)},\alpha_{n'}^{(\nu')\dagger}]=i\delta_{nn'}\delta_{\nu\nu'}$, $[N_\nu,\tilde{Q}_{\nu'}]=i\delta_{\nu,\nu'}$, and $[\theta_{rs},N_{r's'}]=i\delta_{r,r'}\delta_{s,s'}$, we are led to the bosonic commutation relations between the field operators,

$$[\Phi_\nu(x),\Pi_{\nu'}(x')] = i\delta_{\nu\nu'}\delta(x-x'), \quad (14)$$

where we have defined $\Pi_\nu(x)=-\partial_x\Theta_\nu(x)/\pi$. With regard to the zero mode in this bosonization scheme, the eigenvalues of the operators N_{rs} and $\tilde{Q}_\nu$ are obtained from the open boundary conditions. We finally have the TLL Hamiltonian as

$$\mathcal{H}_{\mathrm{TLL}} = \sum_{\nu=\rho,\sigma}\left[\epsilon_\nu N_\nu^2 + \sum_{n=1}^{\infty}\bar{\epsilon}_\nu(n)\left(\alpha_n^{(\nu)\dagger}\alpha_n^{(\nu)}+\frac{1}{2}\right)\right], \quad (15)$$

where

$$v_\nu = \left\{\left[v_F + \frac{g_{4\nu}}{\pi\hbar}\right]^2 - \left[\frac{g_{2\nu}}{\pi\hbar}\right]^2\right\}^{1/2}, \quad (16)$$

are the excitation velocities for the degree of freedom ν,

$$\epsilon_\nu = \hbar\frac{\pi}{2L_d}\frac{v_\nu}{K_\nu} = \frac{\bar{\epsilon}}{2K_\nu}\frac{v_\nu}{v_F} = \frac{\bar{\epsilon}_\nu}{2K_\nu}, \quad (17)$$

and

$$\bar{\epsilon}_\nu(n) = 2K_\nu\epsilon_\nu\cdot n = \bar{\epsilon}_\nu\cdot n, \quad (18)$$

v_F being the Fermi velocity. Note that the first and second terms in Eq. (15) specify the zero mode and the bosonic excitations of the TLL, respectively.

C. Charging energy and electromagnetic environment

In order to describe the electromagnetic energy, we need to determine the phases φ_i canonically conjugate to the charges Q_i of the electrodes (this procedure, known as quantum mechanics with constraints was first discussed by Dirac[23]). In the present case, the condition is that the island charge

$$q = -\sum_{i=1}^{3} Q_i, \quad (19)$$

is constant in time in the absence of tunneling,

$$\sum_{i=1}^{3}\dot{Q}_i = 0. \quad (20)$$

For simplicity, the environmental impedance is taken as $Z_i(\omega)=i\omega L_i$. Given that the Lagrangian of the electromagnetic system is

$$\mathcal{L}_{\mathrm{em}} = \sum_{i=1}^{2}\left\{\frac{L_i}{2}\dot{Q}_i^2 - \frac{Q_i^2}{2C_i} + Q_i(V_i - V_c)\right\} - \frac{Q_3^2}{2C_3} + Q_3(V_3 - V_c) + \lambda\sum_{i=1}^{3}\dot{Q}_i, \quad (21)$$

with a Langrange multiplier λ, we can determine the canonically conjugate phases by

$$\varphi_i = \frac{\partial\mathcal{L}_{\mathrm{em}}}{\partial\dot{Q}_i}. \quad (22)$$

Following standard procedure, the Hamiltonian of the electromagnetic system is

$$\mathcal{H}_{\mathrm{em}} = \sum_{i=1}^{2}\frac{(\varphi_i-\varphi_3)^2}{2L_i} + \sum_{i=1}^{3}\left\{\frac{Q_i^2}{2C_i} - Q_iV_{i,c}\right\}, \quad (23)$$

where $V_{i,\mathrm{c}}$ is the voltage applied to the capacitance C_i,

$$eV_{i,c} = e(V_i - V_c) = \mu_i - \mu_c, \quad (24)$$

expressed in terms of the chemical potentials of the island (μ_c), left electrode (μ_1), right electrode (μ_2), and the gate electrode (μ_3). Now we can quantize $\mathcal{H}_{\mathrm{em}}$ requiring

$$[Q_i,\varphi_{i'}] = i\hbar\delta_{i,i'}, \quad [Q_i,Q_{i'}] = [\varphi_i,\varphi_{i'}] = 0. \quad (25)$$

Note that matrix $\boldsymbol{K}^{1/2}\boldsymbol{Y}_L\boldsymbol{K}^{1/2}$, whose eigenvector is $\boldsymbol{K}^{1/2}\boldsymbol{\lambda}$, diagonalizes $\mathcal{H}_{\mathrm{em}}$ with respect to charges ($\boldsymbol{Q}\to\boldsymbol{Q}'$) and phases ($\boldsymbol{\varphi}\to\boldsymbol{\varphi}'$),[17] where

$$\boldsymbol{\varphi} = \begin{bmatrix}\varphi_1\\ \varphi_2\\ \varphi_3\end{bmatrix}, \quad \boldsymbol{Q} = \begin{bmatrix}Q_1\\ Q_2\\ Q_3\end{bmatrix}, \quad \boldsymbol{V}_c = \begin{bmatrix}V_{1,c}\\ V_{2,c}\\ V_{3,c}\end{bmatrix}, \quad (26)$$

$$\boldsymbol{K} = \begin{bmatrix}\kappa_1 & 0 & 0\\ 0 & \kappa_2 & 0\\ 0 & 0 & \kappa_3\end{bmatrix}, \quad (27)$$

$$\boldsymbol{Y}_L = \begin{bmatrix}\ell_1^{-1} & 0 & -\ell_1^{-1}\\ 0 & \ell_2^{-1} & -\ell_2^{-1}\\ -\ell_1^{-1} & -\ell_2^{-1} & \ell_1^{-1}+\ell_2^{-1}\end{bmatrix}, \quad (28)$$

$$\boldsymbol{\lambda} = (\lambda_{ij}), \quad \lambda_{ij} = \frac{\sqrt{\kappa_i}}{\kappa_i - \ell_i(\omega_j/\omega_L)^2}\left[\sum_{l=1}^{3}\frac{\kappa_l}{\{\kappa_l - \ell_l(\omega_j/\omega_L)^2\}^2}\right]^{-1/2}, \quad (29)$$

with $\kappa_i=C/C_i(i=1,2,3)$, $\ell_j=L_j/L_\Sigma(j=1,2)$, and $\ell_3=0$.

After straightforward calculations, we finally obtain

$$\mathcal{H}_{\mathrm{em}} = \mathcal{H}_{\mathrm{env}} + \mathcal{H}_c, \quad (30)$$

where

$$\mathcal{H}_{\mathrm{env}} = \sum_{j=1}^{2}\left\{\frac{(\omega_j/\omega_L)^2}{2L_\Sigma}\varphi_j'^2 + \frac{Q_j'^2}{2C} - Q_j'V_{j,c}'\right\}, \quad (31)$$

$$\mathcal{H}_c = (q/e - n_c)^2 U. \tag{32}$$

$\mathcal{H}_{\rm env}$ and $\mathcal{H}_c$ describe the electromagnetic environment and island charging effects, respectively. The quantities appearing in Eqs. (31) and (32) are defined as

$$\omega_L = 1/\sqrt{L_\Sigma C}, \tag{33}$$

$$L_\Sigma = L_1 + L_2, \tag{34}$$

$$n_c = -\sum_{j=1}^{3} C_j V_j/e + eV_c/(2U), \tag{35}$$

$$U = e^2/(2C_\Sigma) = E_c C/C_\Sigma, \tag{36}$$

$$C_\Sigma = C_1 + C_2 + C_3, \tag{37}$$

$$C = C_1 C_2/(C_1 + C_2). \tag{38}$$

Here U and n_c are the charging energy of the island and the noninteger charge offset (induced by the voltage bias condition and the charging energy), respectively. Note that n_c includes the chemical potential of the island and should be determined self-consistently by the current continuity condition $I_1+I_2=0$. The electromagnetic environment modes are defined as

$$\left(\frac{\omega_j}{\omega_L}\right)^2 = \frac{1}{2}\left(\frac{\kappa_1+\kappa_3}{\ell_1} + \frac{\kappa_2+\kappa_3}{\ell_2}\right) \pm \sqrt{\frac{1}{4}\left(\frac{\kappa_1}{\ell_1} - \frac{\kappa_2}{\ell_2}\right)^2 + \left[\frac{\kappa_3}{2}\left(\frac{1}{\ell_1}+\frac{1}{\ell_2}\right)\right]^2 + \frac{\kappa_3}{2}\left(\frac{1}{\ell_1}-\frac{1}{\ell_2}\right)\left(\frac{\kappa_1}{\ell_1}-\frac{\kappa_2}{\ell_2}\right)}, \tag{39}$$

where $j=1$ and $j=2$ correspond to the plus and minus signs, respectively. Note that the island charge q is the extra (quantized) charge which describes the deviation from a neutral island. The nonzero eigenvalue of q results only from asymmetric tunneling events through the junctions. Therefore, we can relate q to the zero mode, which describes the change in the number of electrons in the system. Given that

$$q = -\sum_{i=1}^{3} Q_i = -\sqrt{(C_\Sigma/C)}Q_3', \tag{40}$$

and that N_ρ is also the canonical variable conjugate to θ_{rs}, which describes the number of electrons in TLL, we are led to the identities

$$q = -e\sqrt{2}\Delta N_\rho,$$

$$\sqrt{\kappa_i}\lambda_{i3}\varphi_3' = -\frac{\hbar}{e}\theta_{rs}. \tag{41}$$

Here we introduced a different variable ΔN_ρ which describes the change in the charge in the TLL due to tunneling. In order to take charging effect into account, we treat ΔN_ρ as independent variable of N_ρ. Since electron tunneling changes not only the charge but also the spin, it is consistent to introduce ΔN_σ. The eigenstate of ΔN_ν satisfies $\Delta N_\nu|\Delta\bar{N}_\nu\rangle = \Delta\bar{N}_\nu|\Delta\bar{N}_\nu\rangle$ as the charge state of the island $|m\rangle$ satisfies $q|m\rangle = me|m\rangle$. Since $[Q_3', \varphi_3'] = i\hbar$, we have $[\sqrt{\kappa_i}\lambda_{i3}\varphi_3', q] = i\hbar$, and therefore $e^{\pm i\sqrt{\kappa_i}\lambda_{i3}\varphi_3'}|m\rangle = |m\mp 1\rangle$. At this stage, we are led to the Hamiltonian of TLL zero mode fluctuations in the presence of the charging effect,

$$\mathcal{H}_Z = \sum_{\nu=\rho,\sigma} \epsilon_\nu \Delta N_\nu^2 + \mathcal{H}_c = \sum_{\nu=\rho,\sigma} \epsilon_\nu \Delta N_\nu^2 + U(\sqrt{2}\Delta N_\rho + n_c)^2, \tag{42}$$

which specifies the charged state of the TLL island.

D. Tunneling Hamiltonian

According to the discussion above, the tunneling Hamiltonian can be written as

$$\mathcal{H}_T = \sum_{i=1,2} \{\mathcal{H}_T^{(i)} + \mathcal{H}_T^{(i)\dagger}\}, \tag{43}$$

$$\mathcal{H}_T^{(i)} = \sum_{rs} \int d\mathbf{r} \int_0^{L_d} dx T_r^{(i)}(\boldsymbol{r},x) \times e^{i\sum_{j=1}^{2}\sqrt{\kappa_i}\lambda_{ij}e\varphi_j'/\hbar} e^{-i\theta_{rs}} \Psi_s^{(i)\dagger}(\boldsymbol{r})\Psi_{rs}(x),$$
$$= \sum_{\boldsymbol{k}}\sum_{rs} \int_0^{L_d} dx T_{\boldsymbol{k}r}^{(i)}(x) e^{i\sum_{j=1}^{2}\sqrt{\kappa_i}\lambda_{ij}e\varphi_j'/\hbar} e^{-i\theta_{rs}} a_{\boldsymbol{k}s}^{(i)\dagger}\Psi_{rs}(x), \tag{44}$$

where $T_r^{(i)}(\boldsymbol{r},x)$ is the matrix element of tunneling from position x [with chirality $r(=\pm)$] in the island to position $\boldsymbol{r}$ in the ith FL electrode, and

$$T_{\boldsymbol{k}r}^{(i)}(x) = \frac{1}{\sqrt{V^{(i)}}}\int d\boldsymbol{r}\; e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} T_r^{(i)}(\boldsymbol{r},x), \tag{45}$$

is the Fourier transform of $T_r^{(i)}(\boldsymbol{r},x)$ with respect to the Fermi-liquid state $\boldsymbol{k}$. The field operator for the ith FL electrode can be defined as

$$\Psi^{(i)}(\boldsymbol{r}) = \sum_s \Psi_s^{(i)}(\boldsymbol{r}) = \frac{1}{\sqrt{V^{(i)}}}\sum_{\boldsymbol{k}s} a_{\boldsymbol{k}s}^{(i)} e^{i\boldsymbol{k}\cdot\boldsymbol{r}}, \tag{46}$$

where $V^{(i)}$ is the volume of the electrode.

III. TUNNELING CURRENTS

The tunneling current I_i through the ith junction (see Fig. 1) in the lowest order in the tunneling Hamiltonian (see Fig. 2) is given by

$$I_i = i\frac{e}{\hbar}\int_{-\infty}^{+\infty} dt \left\langle \frac{1}{i\hbar}[\mathcal{H}_T^{(i)\dagger}(t),\mathcal{H}_T^{(i)}(0)] \right\rangle \exp\left(i\frac{eV_{i,c}}{\hbar}t \right), \tag{47}$$

where we moved to the grand canonical ensemble, namely,

$$\mathcal{H}_T^{(i)}(t) = e^{itK_0/\hbar}\mathcal{H}_T^{(i)}e^{-itK_0/\hbar}, \quad \langle \cdots \rangle = \frac{\mathrm{Tr}\{\exp(-\beta K_0)\cdots\}}{\mathrm{Tr}\exp(-\beta K_0)},$$

$$K_0 = \mathcal{H}_0 - \sum_{i=1,2,3} \mu_i \int d\boldsymbol{r}_i \Psi^{(i)\dagger}(\boldsymbol{r})\Psi^{(i)}(\boldsymbol{r}) - \mu_c \int_0^{L_d} dx \Psi(x)^\dagger \Psi(x). \tag{48}$$

By calculating the average in Eq. (47), we arrive at the result (see Appendix)

$$I_i = \frac{G_{\mathrm{T0}}^{(i)}}{4e}\int_{-\infty}^{\infty} d\epsilon \tanh\left[\frac{\beta}{2}(\mu_{c,i}-\epsilon)\right]\left(\int_0^{L_d} dx \sum_s \tilde{N}_{\mathrm{TLL}}^{(i)}(\epsilon,x)\right), \tag{49}$$

where $G_{\mathrm{T0}}^{(i)} = (4\pi e^2/\hbar)|T^{(i)}|^2 N_{\mathrm{FL}}(0)N_{\mathrm{1D}}(0)$, with $N_{\mathrm{1D}}(0) = L_d/(\pi\hbar v_F)$ being the density of states per spin in the one-dimensional free-electron system. The normalized local spectral density of TLL $\tilde{N}_{\mathrm{TLL}}^{(i)}(\epsilon,x)$ is given by

$$\tilde{N}_{\mathrm{TLL}}^{(i)}(\epsilon,x) = 2\int_{-\infty}^{\infty}\frac{dt}{2\pi\hbar N_{\mathrm{1D}}(0)}e^{i\epsilon t/\hbar} \times [\mathcal{F}_i^{(\mathrm{env})>}(t)\mathcal{F}_s^{(c)>}(t)G_{+s}(x,x,t) + \mathcal{F}_i^{(\mathrm{env})<}(t)\mathcal{F}_s^{(c)<}(t)G_{+s}(x,x,-t)]. \tag{50}$$

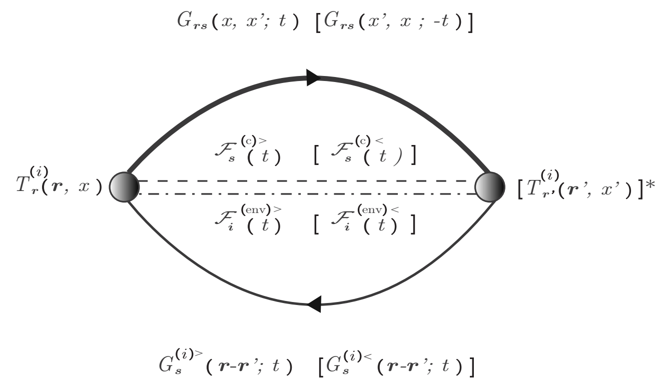

FIG. 2. Diagrammatic representation of the tunneling kernel which corresponds to the statistically averaged quantity in Eq. (47). The bold solid line denotes the exact chiral Green's function of an electron in the TLL. The dotted line and the chain line denote the correlation functions of phases conjugate to quantized (island) charge and continuous (environmental) charge, respectively. The thin solid line denotes the lowest-order Green's functions of an electron in the FL.

The quantities appearing in the kernel in the above expression are the correlation functions of the phases corresponding to the environmental charge $\mathcal{F}_i^{(\mathrm{env})}(t)$, the island charge $\mathcal{F}_s^{(c)}(t)$, and the Green's function of the electron in the TLL, $G_{+s}(x,x,t)$. The environmental and island phase correlation functions are given by

$$\mathcal{F}_i^{(\mathrm{env})\gtrless}(t) = \exp\left\{ \sum_{j=1}^{2} \lambda_{ij}^2 \kappa_i \cdot \frac{E_c}{\hbar\omega_j} \cdot J^{\gtrless}(\omega_j,t) \right\}, \tag{51}$$

$$J^{\gtrless}(\omega,t) = \coth\frac{\beta\hbar\omega}{2}(\cos\omega t - 1) \mp i\sin\omega t, \tag{52}$$

and

$$\mathcal{F}_s^{(c)\gtrless}(t) = \frac{\sum_{\Delta\bar{N}_\rho,\Delta\bar{N}_\sigma=-\infty}^{\infty} e^{-\beta E(\Delta\bar{N}_\rho,\Delta\bar{N}_\sigma)+it/\hbar[\epsilon_\rho(\sqrt{2}\Delta\bar{N}_\rho \mp 1/2)+s\epsilon_\sigma(\sqrt{2}\Delta\bar{N}_\sigma \mp s/2)+U[2(\sqrt{2}\Delta\bar{N}_\rho+\delta n_c)\mp 1]]}}{\sum_{\Delta\bar{N}_\rho,\Delta\bar{N}_\sigma=-\infty}^{\infty} e^{-\beta E(\Delta\bar{N}_\rho,\Delta\bar{N}_\sigma)}}, \tag{53}$$

where

$$E(\Delta\bar{N}_\rho,\Delta\bar{N}_\sigma) = \sum_\nu \epsilon_\nu \Delta\bar{N}_\nu^2 + U(\sqrt{2}\Delta\bar{N}_\rho + n_c)^2, \tag{54}$$

$$\delta n_c = n_c - [n_c + 1/2], \tag{55}$$

and $[\cdots]$ is the Gauss symbol. The exact chiral Green's function[22] is given by

$$G_{+s}(x,y,t) = \langle \bar{\psi}^{\dagger}_{+s}(x,t)\bar{\psi}_{+s}(y,0)\rangle = \left\langle \frac{1}{\sqrt{2\pi\alpha}} e^{-i\Phi_{+s}(x,t)} \frac{1}{\sqrt{2\pi\alpha}} e^{i\Phi_{+s}(y,0)} \right\rangle$$
$$= \frac{1}{2\pi\alpha} e^{-i\pi/2L(x-y)} H(x,y,t) \prod_{\nu=\rho,\sigma} \Bigg\{ [F_\nu(v_\nu t - x + y)]^{-1/2(a_\nu+1/2)} [F_\nu(v_\nu t + x - y)]^{-1/2(a_\nu-1/2)}$$
$$\times \left[\frac{|F_\nu(2x)F_\nu(2y)|}{F_\nu(v_\nu t - x - y)F_\nu(v_\nu t + x + y)} \right]^{b_\nu/2} \Bigg\}, \tag{56}$$

with

$$a_\nu = \frac{1}{4}\left(\frac{1}{K_\nu} + K_\nu\right), \quad b_\nu = \frac{1}{4}\left(\frac{1}{K_\nu} - K_\nu\right). \tag{57}$$

The zero mode contribution is

$$H(x,y,t) = \prod_{\nu=\rho,\sigma} \langle e^{s^{\delta_{\nu,\sigma}}(iu_\nu\sqrt{2}N_\nu)} \rangle$$
$$= \frac{\vartheta_2(u_\rho|\tau_\rho)\vartheta_2(u_\sigma|\tau_\sigma) + \vartheta_3(u_\rho|\tau_\rho)\vartheta_3(u_\sigma|\tau_\sigma)}{\vartheta_2(0|\tau_\rho)\vartheta_2(0|\tau_\sigma) + \vartheta_3(0|\tau_\rho)\vartheta_3(0|\tau_\sigma)}, \tag{58}$$

where $\vartheta_n(u|\tau)$ are theta functions, and $u_\nu = -\frac{\pi}{2L_d}(\frac{v_\nu}{K_\nu}t + x - y)$ and $\tau_\nu = i2\beta\epsilon_\nu/\pi$. Function $F_\nu(z)$ appearing in Eq. (56) is given by

$$F_\nu(z) = i\frac{2L_d}{\pi\alpha}\sin\frac{\pi z}{2L_d}\prod_{k=1}^{\infty}\left[1 + \left(\frac{\sin\frac{\pi z}{2L_d}}{\sinh\frac{k\pi v_\nu\beta\hbar}{2L_d}}\right)^2\right]$$
$$= i\frac{L_d}{\pi\alpha}\left[\eta\left(\frac{iv_\nu\beta\hbar}{2L_d}\right)\right]^{-3}\vartheta_1\left(\frac{\pi z}{2L_d}\middle|\frac{iv_\nu\beta\hbar}{2L_d}\right), \tag{59}$$

where $\eta(\tau)$ is the Dedekind eta function. Note that $\tilde{N}^{(i)}_{\mathrm{TLL}}(\epsilon,x)$ is modified by the open boundary condition as well as the charging effect. This is essential when discussing TLL behavior in the Coulomb blockade regime. The two cases corresponding to bulk (with translational invariance) and edge contacts (without translational invariance) are obtained by considering the limits of $x/v_\nu t \gg 1$ and $x/v_\nu t \ll 1$, respectively, in the above expression for the chiral Green's function.

Since I_i is a function of chemical potentials of ith FL electrode μ_i and TLL electrode μ_c, the tunneling current of the system I is obtained from

$$I = \frac{1}{2}(I_1 - I_2) \tag{60}$$

under the current continuity condition

$$\sum_{i=1}^{2} I_i = 0. \tag{61}$$

We first consider the effect of the environment fluctuations on an infinite system. When deriving the Hamiltonian (30) we have, for simplicity, considered a purely inductive environment but the approach is also applicable to arbitrary linear circuit elements. In this case, the derivation can be made along the lines presented above but with an infinite number of oscillators whose spectral density is chosen to reproduce Johnson–Nyquist correlations (see, for example, review articles[24] and references therein). The usual experimental setup includes a dissipative environment which is Ohmic at low frequencies $Z_i(\omega)=R_i$ ($\omega\to 0$). In this case Eq. (51) is replaced by $\exp[J_i^{\gtrless}(t)]$, where

$$\Re[J_i^{\gtrless}(t)] \sim -\frac{2}{R_K}\left(\frac{C_j}{C_\Sigma}\right)^2\left[\left(1+\frac{C_3}{C_j}\right)^2 R_i + R_j\right]$$
$$\times \ln\left(\frac{\hbar\omega_o^{(i)}\beta}{\pi}\sinh\frac{\pi|t|}{\hbar\beta}\right), \tag{62}$$

for large t, where i, $j=1,2(i\neq j)$, and where $\omega_o^{(i)}$ is the environment-dependent cut-off frequency given by

$$\omega_o^{(i)} = \frac{(C_j + C_3)C_\Sigma}{C_i C_j^2[(1 + C_3/Cj)^2 R_i + R_j]}. \tag{63}$$

From Eq. (59), we have

$$F_\nu(v_\nu t) \to i\frac{v_\nu\beta\hbar}{\alpha\pi}\sinh\left(\frac{\pi t}{\beta\hbar}\right), \quad (L_d\to\infty). \tag{64}$$

For simplicity, let us consider the symmetric system, $R_1 = R_2 = R/2$, $C_1 = C_2 = 2C$, $V_1 = -V_2 = V/2$, and $V_3 = 0$. By substituting Eq. (62) into Eq. (51) and Eq. (64) into Eq. (56), and by performing a Fourier transform in Eq. (50), we get the conductance from Eq. (49), in the limit $\beta\to\infty$, near the CB region ($V \gtrsim e^2/C_\Sigma$),

$$G = \frac{G_{\mathrm{T0}}(\alpha\omega_o)^{\alpha_c}v_F}{4\Gamma(\tilde{\alpha}_c+1)v_\rho^{\beta_\rho}v_\sigma^{\beta_\sigma}}\left(\frac{eV - e^2/C_\Sigma}{2\hbar\omega_o}\right)^{\tilde{\alpha}_c}, \tag{65}$$

$$\tilde{\alpha}_c = \alpha_c + c\zeta, \quad \alpha_c = \sum_\nu \beta_\nu - 1, \quad \beta_\nu = \begin{cases} a_\nu & \text{(bulk)} \\ a_\nu + b_\nu & \text{(edge)}, \end{cases} \tag{66}$$

where the environmental parameters are $\zeta = R/R_K$ and $c = [(2C+C_3)^2 + (2C)^2]/(4C+C_3)^2$. By increasing the voltage further away from the CB region ($V \gg e^2/C_\Sigma$), we obtain the above formula with $\tilde{\alpha}_c \to \alpha_c$, and the conductance approaches the power law with the usual TLL exponent.

In the case of the finite system size, for $\beta\to\infty$, we obtain near the CB region ($V \gtrsim e^2/C_\Sigma$),

$$G = \frac{G_{T0}2^{\gamma_c-1}}{\Gamma(c\zeta)N_{1D}(0)\hbar\omega_o}\left(\frac{\alpha\pi}{L_d}\right)^{\alpha_c}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\mathcal{F}(n,m), \qquad (67)$$

$$\mathcal{F}(n,m) = \frac{1}{n\,!\,m!}\frac{\Gamma(n+\beta_\rho)\Gamma(m+\beta_\sigma)}{\Gamma(\beta_\rho)\Gamma(\beta_\sigma)}f(V)^{c\zeta-1}e^{-f(V)}\Theta[f(V)], \qquad (68)$$

$$f(V) = \frac{1}{2\hbar\omega_o}\left[eV - 2U - \bar{\epsilon}\sum_\nu \frac{2\beta_\nu K_\nu + 1}{2K_\nu^2} - 2\bar{\epsilon}\left(\frac{n}{K_\rho}+\frac{m}{K_\sigma}\right)\right], \qquad (69)$$

$$\gamma_c = \begin{cases} 0 & \text{(bulk)} \\ b_\rho + b_\sigma & \text{(edge)}. \end{cases} \qquad (70)$$

The above formula shows that by increasing the electron-electron interaction parameter, the Coulomb gap region extends due to the 1D nature of the island. For a large island length, the indexes n and m can be considered as continuous due to the small energy gap $\bar{\epsilon}$ that these indexes multiply. We recover formula (65) by using Stirling's approximation and performing the integrations. For a short island length, we can retain only the $\mathcal{F}(0,0)$ term in Eq. (67) due to the unit step function in Eq. (68). Away from the CB region, instead of $\mathcal{F}$, we have a set of delta functions which—after integration—reproduce the corresponding long length limit of formula (65).

We consider now the conductance at zero voltage of a finite system in the high-temperature regime ($\epsilon_\nu\beta \ll 1$). By using the asymptotic behavior of elliptic theta functions for $\tau\to 0$,[25] $\vartheta_1(u|e^{-\tau}) \sim 2\sqrt{\frac{\pi}{\tau}}\exp(-\frac{\pi^2+(2u)^2}{4\tau})\sinh(\frac{\pi u}{\tau})$, and $\vartheta_j(u|e^{-\tau}) \sim \sqrt{\frac{\pi}{\tau}}\exp(-\frac{u^2}{\tau})[1-(-1)^j 2\exp(-\frac{\pi^2}{\tau})\cosh(\frac{2\pi u}{\tau})]$ for $j=2,3$, the chiral Green's function can be written as

$$G_{+s}(x,x,t) = \frac{2^{\gamma_c-1}}{\pi\alpha}\exp\left(-\frac{\bar{\epsilon}t^2}{\beta\hbar^2}\sum_\nu\frac{1-2\beta_\nu K_\nu^2}{4K_\nu^3}\right)\left(\frac{\alpha\pi^2}{L_d\bar{\epsilon}\beta}\right)^{\Sigma_\nu\beta_\nu} \times K_\rho^{\beta_\rho}K_\sigma^{\beta_\sigma}\left[i\sinh\left(\frac{\pi t}{\hbar\beta}\right)\right]^{\Sigma_\nu\beta_\nu}. \qquad (71)$$

By using Eq. (62) in $\mathcal{F}_i^{(\text{env})\gtrless}(t)$, approximating the summations in $\mathcal{F}_s^{(c)\gtrless}(t)$ by integrals and after substituting in Eqs. (50) and (49) we obtain the conductance for $\hbar\omega_o\beta \gg 1$,

$$G = G_{T0}\frac{2^{\gamma_c}}{\pi}\left(\frac{\alpha\pi^2}{L_d}\right)^{\alpha_c}K_\rho^{\beta_\rho}K_\sigma^{\beta_\sigma}\left(\frac{\pi\bar{\epsilon}}{\hbar\omega_o}\right)^{c\zeta}(\bar{\epsilon}\beta)^{-\tilde{\alpha}_c}\mathcal{I}(\tilde{\alpha}_c+1), \qquad (72)$$

$$\mathcal{I}(a) = \int_0^\infty dw\ e^{-\beta U w^2}\cos\left(\frac{\pi}{2}a + \beta U w\right)\frac{w^a-(\sinh w)^a}{(\sinh w)^{a+1}w^{a-1}}. \qquad (73)$$

For higher temperatures $\beta U \ll 1$, where the Coulomb blockade is washed out, we obtain the above formula (72) with replacements $c\zeta\to 0$ and $\tilde{\alpha}_c\to\alpha_c$, which is the usual TLL power law.[22]

On the other hand, in the low-temperature regime where ($\epsilon_\nu\beta \gg 1$), even without any external impedance, the conductance is exponentially suppressed,

$$G = G_{T0}2^{\gamma_c}\left(\frac{\pi\alpha}{L_d}\right)^{\alpha_c}\bar{\epsilon}\beta\exp\left[-\left(\bar{\epsilon}\beta\sum_\nu\frac{2\beta_\nu K_\nu+1}{4K_\nu^2}+\beta U\right)\right]. \qquad (74)$$

The conductance decreases monotonously with increasing electron-electron interaction similar to the effect of increasing the environment impedance if the island was a FL.[26]

IV. SUMMARY

Analytic expressions for the tunnel current have been derived for the electron transport in a C-SET with a TLL island. For an infinite system and a general electromagnetic environment with dissipative Ohmic impedance at low frequencies, the conductance shows power-law behavior as a function of voltage at zero temperature. Near the CB threshold voltage, this power [$\tilde{\alpha}_c$ in Eq. (66)] differs from the usual TLL power (α_c) as a result of the electromagnetic environment. For high voltages the environmental effects disappear and the usual TLL power-law behavior results with the voltage offset due to the CB of the island. For a finite system in the same limit of zero temperature, we obtained the analytic expression for the conductance versus voltage near the CB region. This shows that for a short length case, the power depends only on the environment.

Zero voltage conductance of a finite system as a function of temperature is given by the expressions (72) and (73) (for $\epsilon_\nu\beta \ll 1$), with power being modified by the environment for low temperatures, compared to the environmental cut-off frequency ($\hbar\omega_o\beta \gg 1$), and the usual TLL power law is recovered at higher temperatures ($\beta U \ll 1$). Conductance is strongly suppressed at low temperatures ($\epsilon_\nu\beta \gg 1$).

The analogous effect of the correlation functions of bosonic modes of the environment and the charge and spin excitations on conductance is significant for SET devices, such as those used in metrology, particularly with regard to the SI. The accuracy of SET devices is limited by cotunneling and its effect can be reduced by environmental impedance or by increasing the electron-electron interaction in the TLL. Therefore, a pump device consisting of TLL islands (or combination of TLL and FL islands) could be used for increased accuracy.

APPENDIX

The expression for tunneling current through ith junction in the lowest order with respect to the tunnel Hamiltonian is derived by evaluating Eq. (47),

$$I_i = i\frac{e}{\hbar}\int_{-\infty}^{+\infty}dt\left\langle\frac{1}{i\hbar}[\mathcal{H}_T^{(i)\dagger}(t),\mathcal{H}_T^{(i)}(0)]\right\rangle\exp\left(i\frac{eV_{i,c}}{\hbar}t\right). \qquad (A1)$$

Note that the tunneling kernel becomes

$$X_i(t) = \langle[\mathcal{H}_T^{(i)\dagger}(t),\mathcal{H}_T^{(i)}(0)]\rangle = \mathcal{F}_i^{(\mathrm{env})>}(t)\mathcal{F}_s^{(c)>}\alpha_T^{(i)>}(t) - \mathcal{F}_i^{(\mathrm{env})<}(t)\mathcal{F}_s^{(c)<}\alpha_T^{(i)<}(t), \quad \text{(A2)}$$

with various quantities defined as follows:

$$\mathcal{F}_i^{(\mathrm{env})>}(t) = \left\langle \exp\left\{-i\frac{e}{\hbar}\sum_{j=1}^{2}\sqrt{\kappa_i}\lambda_{ij}\varphi_j'(t)\right\} \times \exp\left\{i\frac{e}{\hbar}\sum_{j=1}^{2}\sqrt{\kappa_i}\lambda_{ij}\varphi_j'(0)\right\}\right\rangle, \quad \text{(A3)}$$

$$\mathcal{F}_i^{(\mathrm{env})<}(t) = \left\langle \exp\left\{i\frac{e}{\hbar}\sum_{j=1}^{2}\sqrt{\kappa_i}\lambda_{ij}\varphi_j'(0)\right\} \times \exp\left\{-i\frac{e}{\hbar}\sum_{j=1}^{2}\sqrt{\kappa_i}\lambda_{ij}\varphi_j'(t)\right\}\right\rangle, \quad \text{(A4)}$$

$$\mathcal{F}_s^{(c)>} = \langle e^{i\theta_{rs}(t)}e^{-i\theta_{rs}(0)}\rangle, \quad \text{(A5)}$$

$$\mathcal{F}_s^{(c)<} = \langle e^{-i\theta_{rs}(0)}e^{i\theta_{rs}(t)}\rangle, \quad \text{(A6)}$$

$$\alpha_T^{(i)>}(t) = \sum_s\sum_{rr'}\int_0^{L_d}\int_0^{L_d}dx dx'\sum_{\boldsymbol{k}}T_{\boldsymbol{k}r}^{(i)*}(x)T_{\boldsymbol{k}r'}^{(i)}(x')e^{-ik_F(rx-r'x')} \times G_i^{(\mathrm{FL})>}(\boldsymbol{k},t)\langle\eta_{rs}^\dagger\eta_{r's'}\bar{\psi}_{rs}^\dagger(x,t)\bar{\psi}_{r's}(x',0)\rangle, \quad \text{(A7)}$$

$$\alpha_T^{(i)<}(t) = \sum_s\sum_{rr'}\int_0^{L_d}\int_0^{L_d}dx dx'\sum_{\boldsymbol{k}}T_{\boldsymbol{k}r}^{(i)*}(x)T_{\boldsymbol{k}r'}^{(i)}(x')e^{-ik_F(rx-r'x')} \times G_i^{(\mathrm{FL})<}(\boldsymbol{k},t)\langle\eta_{r's'}\eta_{rs}^\dagger\bar{\psi}_{r's}(x',0)\bar{\psi}_{rs}^\dagger(x,t)\rangle, \quad \text{(A8)}$$

$$\begin{pmatrix} G_i^{(\mathrm{FL})>}(\boldsymbol{k},t) \\ G_i^{(\mathrm{FL})<}(\boldsymbol{k},t)\end{pmatrix} = \begin{pmatrix}\langle a_{\boldsymbol{k}s}^{(i)}(t)a_{\boldsymbol{k}s}^{(i)\dagger}(0)\rangle \\ \langle a_{\boldsymbol{k}s}^{(i)\dagger}(0)a_{\boldsymbol{k}s}^{(i)}(t)\rangle\end{pmatrix}, \quad \text{(A9)}$$

and

$$\bar{\psi}_{rs}(x) = \eta_{rs}\psi_{rs}(x) = \frac{1}{\sqrt{2\pi\alpha}}e^{i\Phi_{rs}(x)}. \quad \text{(A10)}$$

The correlation functions of phases $\mathcal{F}_i^{(\mathrm{env})}(t)$ and $\mathcal{F}_s^{(c)}(t)$ describe the electromagnetic environment effect and the charging effect in the TLL island, respectively. In the case of $\mathcal{F}_i^{(\mathrm{env})}(t)$, it is standard procedure to take the trace of boson correlation function of this type to give Eq. (51). However, in the case of $\mathcal{F}_s^{(c)}(t)$ and bearing in mind that

$$e^{\mp i\theta_{rs}}|\Delta\bar{N}_\nu\rangle = \left|\Delta\bar{N}_\nu \mp \frac{s^{\delta_{\nu,\sigma}}}{\sqrt{2}}\right\rangle, \quad \text{(A11)}$$

for the eigenstate of $q=-e\sqrt{2}\Delta N_\rho$ and noting that the trace should be taken with respect to $\mathcal{H}_Z$, we obtain Eq. (53).

In deriving $\alpha_T^{(i)\gtreqless}(t)$, $\boldsymbol{k}$ dependence of $T_{\boldsymbol{k}r}^{(i)}(x)$ can be ignored as usual. Furthermore, we assume

$$T_{\boldsymbol{k}r}^{(i)*}(x)T_{\boldsymbol{k}r'}^{(i)}(x') \sim |T^{(i)}|^2\delta(x-x'), \quad \text{(A12)}$$

since $T_{\boldsymbol{k}r}^{(i)*}(x)T_{\boldsymbol{k}r'}^{(i)}(x')$ is dominant when $x'\sim x\sim 0$ (for $i=1$) or $x'\sim x\sim L_d$ (for $i=2$). After carrying out a summation over $\boldsymbol{k}$, we arrive at

$$\alpha_T^{(i)\gtreqless}(t) = -2i\hbar N_{\mathrm{FL}}^{(i)}(0)|T^{(i)}|^2\frac{\pi/\beta\hbar}{\sinh(\pi t/\beta\hbar)} \times \sum_s\int_0^{L_d}dx\ G_{+s}(x,x,\pm t), \quad \text{(A13)}$$

where $N_{\mathrm{FL}}^{(i)}(0)$ is the density of states of ith FL electrode at the Fermi level. When deriving Eq. (A13) we took $r'=r$ only, since the factor $e^{-ik_F(r-r')x}$ averages out the integrand over several lattice sites. Using expressions derived above, Eq. (A2) becomes

$$X_i(t) = -i\hbar N_{\mathrm{FL}}^{(i)}(0)|T^{(i)}|^2\frac{\pi/\beta\hbar}{\sinh(\pi t/\beta\hbar)} \times\int_{-\infty}^{\infty}d\epsilon e^{-i\epsilon t/\hbar}\left[\int_0^{L_d}dx\sum_s\tilde{N}_s^{(i)}(\epsilon,x)\right], \quad \text{(A14)}$$

where we defined the effective local spectral density as

$$\tilde{N}_s^{(i)}(\epsilon,x) = 2\int_{-\infty}^{\infty}\frac{dt}{2\pi\hbar}e^{i\epsilon t/\hbar}[\mathcal{F}_i^{(\mathrm{env})>}(t)\mathcal{F}_s^{(c)>}(t)G_{+s}(x,x,t) + \mathcal{F}_i^{(\mathrm{env})<}(t)\mathcal{F}_s^{(c)<}(t)G_{+s}(x,x,-t)]. \quad \text{(A15)}$$

By substituting Eq. (A14) into Eq. (A1), we obtain the tunneling current through junction i

$$I_i = G_{\mathrm{T0}}^{(i)}\frac{1}{4e}\int_{-\infty}^{\infty}d\epsilon\tanh\left[\frac{\beta}{2}(\mu_{c,i}-\epsilon)\right]\left(\int_0^{L_d}dx\sum_s\tilde{N}_{\mathrm{TLL}}^{(i)}(\epsilon,x)\right), \quad \text{(A16)}$$

where $G_{\mathrm{T0}}^{(i)}=(4\pi e^2/\hbar)|T^{(i)}|^2N_{\mathrm{FL}}^{(i)}(0)N_{\mathrm{1D}}(0)$, $N_{\mathrm{1D}}(0)=L_d/(\pi\hbar v_F)$, and

$$\tilde{N}_{\mathrm{TLL}}^{(i)}(\epsilon,x) = \frac{\tilde{N}_s^{(i)}(\epsilon,x)}{N_{\mathrm{1D}}(0)}, \quad \text{(A17)}$$

is the normalized effective local spectral density of the TLL island.

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